## Soil Mechanics II and Exercises [Final Exam]

July 31, 2024 (Wed.) 10:00–12:00 Kyotsu 1 lecture room

## Attention:

- The exam consists of five questions for which you are provided with five answer sheets. Write down your name and student ID number on every answer sheet. Use one answer sheet per question and answer them in sequence, starting from [Question 1]. If the front page of an answer sheet is insufficient to complete your answer, use **the back page of the same answer sheet** after clearly indicating your intent.
- In addition to personal writing instruments, use of non-programmable calculators and rulers are permitted. However, programmable calculators and calculator functions of mobile phones are prohibited.
- Wherever necessary, specify the units in your answers.
- Any attempt at cheating on the exam will result in failed credit of the course and serious penalties.

[Question 1] The ground shown in Figure 1-1 consists of a 8.0 m thick layer of homogeneous clay deposited above impermeable rock. The clay is normally consolidated. A constant groundwater level is located at the ground surface. A sample was taken from the clay layer for consolidation test. The compression index,  $C_c$ , and initial void ratio,  $e_0$ , of the clay sample are found to be 0.75 and 0.8, respectively. It was also found that the time required for 50% consolidation of a 20 mm-thick specimen (drained on both sides) in the laboratory is 2 min 20 sec. A uniformly distributed rectangular shape surcharge is to be placed at the ground surface. The surcharge has a dimension of 4.0 m × 8.0 m with magnitude  $q = 30 \text{ kN/m}^2$  as illustrated in Figure 1-2. Assume the compression of soil and the water flow are one-dimensional and ignore immediate settlement, the stress increment in the clay layer due to the construction of the structure can be determined by the elasticity theory. Consider the properties of the clay sample to be representative values of the whole clay layer. Answer the questions below. You can use Table 1-1 and the chart given in Figure 1-3 for your calculation. The unit weight of water is 9.8 kN/m<sup>3</sup>.

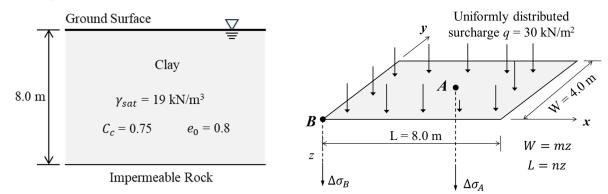


Figure 1-1 Ground profile.

Figure 1-2 Surcharge at ground surface.

U (%)	$T_{v}$	U (%)	$T_{\nu}$	U (%)	$T_{v}$	U (%)	$T_{\nu}$	U (%)	$T_{v}$
0	0.000	20	0.031	40	0.126	60	0.286	80	0.567
5	0.002	25	0.049	45	0.159	65	0.340	85	0.684
10	0.008	30	0.071	50	0.197	70	0.403	90	0.848
15	0.018	35	0.096	55	0.239	75	0.477	95	1.129
								100	$\infty$

Table 1-1 Relationship between average degree of consolidation (U) and time factor  $(T_v)$ .

(1) Calculate the effective stress  $\sigma'_p$  at the middle of the clay layer before placement of the surcharge.

- (2) After placement of the surcharge, calculate the vertical stress increase,  $\Delta \sigma_A$ , at the middle of the clay layer underneath the center point *A* of the surcharge area.
- (3) After placement of the surcharge, calculate the vertical stress increase,  $\Delta \sigma_B$ , at the middle of the clay layer underneath the corner point *B* of the surcharge area.
- (4) Based on the  $\Delta \sigma_A$  calculated in (2), find the expected primary consolidation settlement of the clay layer at the center of the surcharge area. Use  $\sigma'_p$  calculated in (1) as the existing effective stress in the clay.
- (5) Find the coefficient of consolidation,  $c_v$ , of the clay based on the laboratory test result.
- (6) Calculate the consolidation settlement of the clay one year after placement of the surcharge.
- (7) Suggest two possible ways to accelerate the consolidation of the clay, and briefly explain why they can accelerate the consolidation.

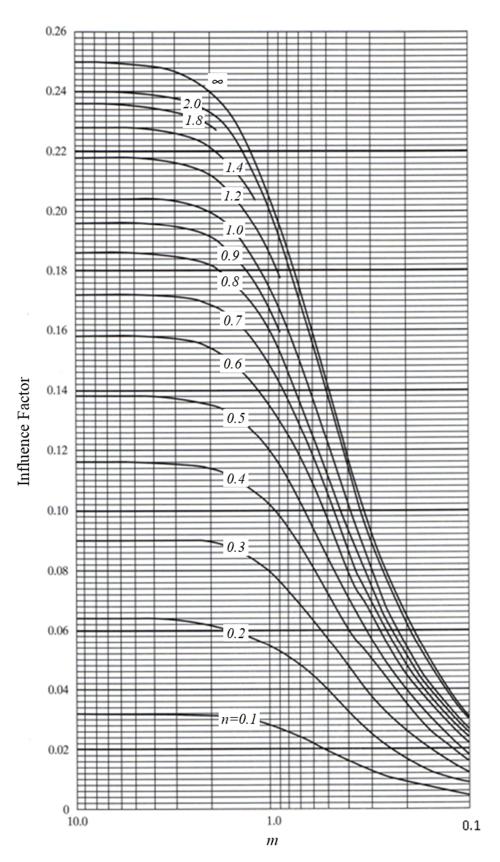


Figure 1-3 Influence factor under corner of rectangular footing.

[Question 2] To determine the active earth pressure acting on the retaining wall shown in Figure 2, answer the following questions. Assume that the back of the retaining wall is vertical and friction along the wall is negligible, and a unit width of the wall perpendicular to the page is considered.

- Describe the theoretical background of Coulomb's and Rankine's earth pressure theories, which are representative methods for calculating earth pressure, in 100-200 words each.
- (2) The backfill behind the retaining wall shown in the figure is compacted unsaturated sand. Given the sand's cohesion as c [kN/m<sup>2</sup>], internal friction angle as  $\phi$  [°], and wet unit weight as  $\gamma_t$  [kN/m<sup>3</sup>], we want to determine the resultant active earth pressure acting on the entire retaining wall per unit width perpendicular to the page using the Rankine's earth pressure theory. First, calculate the horizontal earth pressure  $\sigma_{ha}$  [kN/m<sup>2</sup>] at a depth z [m]. Note that the groundwater table is located sufficiently deep below the base of the retaining wall.
- (3) Derive the formula to calculate the resultant active earth pressure,  $P_a$ , acting on the retaining wall, considering the height of the retaining wall as H [m]. Also, determine the value of  $P_a$  when c = 12 kN/m<sup>2</sup>,  $\phi = 30^{\circ}$ ,  $\gamma_t = 15$  kN/m<sup>3</sup>, and H = 6 m.
- (4) Due to insufficient drainage behind the retaining wall, rainfall water infiltrated the ground and formed a groundwater level at the surface of the backfill. The backfill is now saturated, with a cohesion of c' = 0 kN/m<sup>2</sup>, an internal friction angle of  $\phi' = 30^{\circ}$ , and a saturated unit weight of  $\gamma_{sat} = 18$  kN/m<sup>3</sup>. Calculate the resultant active earth pressure,  $P_a$ , acting on the retaining wall. For simplicity, the unit weight of water can be approximated by  $\gamma_w = 10$  kN/m<sup>3</sup>.

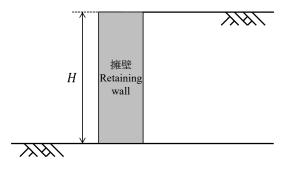


Figure 2

[Question 3] Answer the following questions regarding bearing capacity of shallow foundation.

(1) In the design of shallow foundations, the following generalized formula is used as the ultimate bearing capacity equation considering the geometry of the foundation:

$$q_u = \frac{Q_u}{A} = \alpha c N_c + \frac{1}{2} \beta \gamma B N_\gamma + \gamma D_f N_q \qquad [Eq. 1]$$

where  $Q_u$  is the ultimate load, A is the base area of the foundation, c is the cohesion,  $N_c, N_\gamma, N_q$  are bearing capacity factors, B is the width of the footing,  $D_f$  is the depth of footing,  $\gamma$  is the unit weight of soil, and  $\alpha, \beta$ are coefficients related to the shape at bottom of the footing. Briefly explain the meaning of each term in the above equation.

- (2) Considering the situation as shown in Figure 3(a), where a circular footing with the diameter *B* is placed on a horizontal sandy sediment below water surface. Let  $q_{u1}$  be the ultimate bearing capacity in this case, derive the formula for  $q_{u1}$  from [Eq. 1]. The sediment is completely saturated, and the cohesion of the sandy soil can be assumed to be c = 0. Let  $\gamma_{sat}$  be the saturated unit weight of soil and  $\gamma_w$  be the unit weight of water, and the submerged unit weight of soil is defined as  $\gamma' = \gamma_{sat} \gamma_w$ .
- (3) Then, the surface of the sandy sediments is lowered, and the footing depth is reduced to  $D_f/2$ , as shown in Figure 3(b). Let  $q_{u2}$  be the ultimate bearing capacity in this case, express  $q_{u2}$  in a form containing  $q_{u1}$ . Note that the sediment surface is assumed to remain horizontal after the ground surface is lowered and the soil strength constants do not vary with the footing depth.
- (4) In the situation in (3), when  $q_{u2} = (2/3) \times q_{u1}$ , express the ultimate bearing capacity  $q_{u1}$ , which is the one before the ground surface was lowered, using  $\gamma', D_f, N_q$ .
- (5) In the situation in (3), derive the allowable bearing capacity,  $q_a$ , with the factor of safety, F.
- (6) Even after the footing depth decreases and the bearing capacity decreases, it is required to maintain a factor of safety of F = 2. Let Q be the design load, derive the conditions that the initial footing depth  $D_f$  must satisfy in order to safely support it, using B, Q,  $N_q$ ,  $\gamma'$ .
- (7) In general, the bearing capacity factors  $N_c$ ,  $N_\gamma$ ,  $N_q$  are expressed as monotonically increasing functions of the internal friction angle  $\phi$ . Discuss how the required footing depth changes with the change in internal friction angle based on the results of (6).

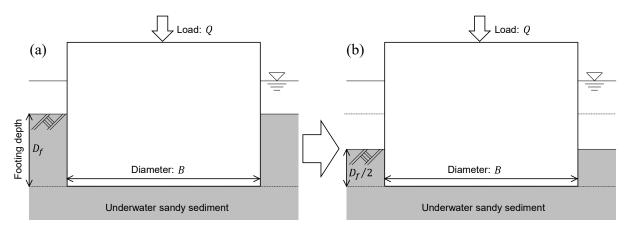
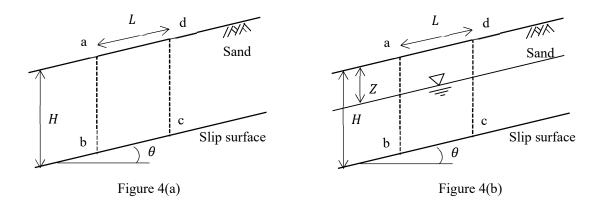


Figure 3

[Question 4] Answer the following questions.

Consider the stability of an infinite slope of sand as shown in the figures.  $\theta$ : angle between the slip surface and horizontal plane, *H*: depth of the slip surface,  $\gamma_t$ : wet unit weight of sand,  $\gamma_{sat}$ : saturated unit weight of sand,  $\gamma_w$ : unit weight of water, *c*: cohesion of wet sand.  $\phi$ : internal friction angle of wet sand, *c'*: effective stress-based cohesion,  $\phi'$ : effective stress-based internal friction angle.



- (1) Considering the force acting on the sliding soil mass abcd (unit depth) of length L along the slope shown in Figure 4(a), calculate the factor of safety  $F_s$  against sliding for the slip surface (at vertical depth H). Here, assume that there is no groundwater and that the sand is in a wet state.
- (1-1) Show the weight of the sliding mass using the given symbols.
- (1-2) Show the normal force in the direction normal to the slip surface using the given symbols.
- (1-3) Show the driving force parallel to the slip surface using the given symbols.
- (1-4) Show the resistance force parallel to the slip surface using the given symbols.
- (1-5) Show the factor of safety against sliding using the given symbols.
- (2) As shown in Figure 4(b), rainfall formed a groundwater level (vertical depth Z) parallel to the slope. Considering the force acting on a sliding mass abcd (unit depth) of length L along the slope, calculate the factor of safety  $F_s$  against sliding for the slip surface (at vertical depth H). Here, the sand below the groundwater level is saturated.
- (2-1) Show the weight of the sliding mass using the given symbols.
- (2-2) Show the normal force in the direction normal to the slip surface using the given symbols.
- (2-3) Show the driving force parallel to the slip surface using the given symbols.
- (2-4) Show the resistance force parallel to the slip surface using the given symbols.
- (2-5) Show the factor of safety against sliding using the given symbols.
- (3) Based on the results of (1) and (2), explain the change in the factor of safety against sliding of a slope due to rainfall.
- (4) Explain how to determine the cohesions and internal friction angles for the saturated sand in (2) through laboratory tests.

## [Question 5] Answer the following questions.

- (1) Answer the following questions regarding liquefaction.
- (1-1) Figure 5-1 shows (a) the effective stress path and (b) the relationship between shear strain and shear stress obtained from an undrained cyclic hollow torsional shear test on sand. Explain cyclic mobility using this diagram.
- (1-2) Explain the definition of the factor of safety  $F_L$  against liquefaction and briefly explain how to calculate it.
- (2) Answer the following questions regarding ground vibration.
- (2-1) Based on the z-directional propagation of SH waves in the two-layer soil model shown in Figure 5-2, derive the impedance ratio of the two layers. Densities of layer 1 and layer 2 are  $\rho_1$ ,  $\rho_2$ , respectively, and shear wave velocities of layer 1 and layer 2 are  $V_1$ ,  $V_2$ , respectively. Note that the horizontal displacement of the two layers can be calculated using the following equations.

Horizontal displacement of layer 1:  $u_1 = g_1 \left( t - \frac{z}{v_1} \right) = B \exp \left( i \omega \left( t - \frac{z}{v_1} \right) \right)$ 

Horizontal displacement of layer 2:  $u_2 = f_1 \left( t - \frac{z}{v_2} \right) + f_2 \left( t + \frac{z}{v_2} \right) = \exp \left( i\omega \left( t - \frac{z}{v_2} \right) \right) + A \exp \left( i\omega \left( t + \frac{z}{v_2} \right) \right)$ 

(2-2) Consider the earthquake response of a surface layer situated above a rigid foundation. If the thickness of the surface layer is H = 10 m and the shear wave velocity of the surface layer is  $V_s = 100$  m/s, determine the dominant period at which this surface layer is most susceptible to shaking.

